Utility Function Slides

Econ 360

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Learning Goals/Outcomes

- Describe a utility function and key properties of a utility function.
- Explain the relationship between utility functions and indifference curves.
- Derive the Marginal Rate of Substitution (MRS) from an indifference curve graphically and with math.

Where We Are

- An indifference curve represents a consumer's preferences.
- We know how to use indifference curves to rank and compare any two bundles.
- We know how to identify the better set and worse set for any bundle.
- We know the slope of the indifference curve (MRS) represents a consumer's willingness to trade one commodity for another, and that the MRS is not necessarily constant along an indifference curve.

Utility Functions: The Big Idea

- We want to compare any two bundles.
- We *could* draw an indifference curve through one bundle and see if the other bundle was on the same indifference curve, a lower indifference curve, or a higher indifference curve.
- But that feels a bit clunky, surely there is an easier way.
- Instead we will "score" each bundle based on some **utility function** $U(\cdot)$.
- Then we can simply say the bundle with the higher score is the one the consumer prefers, and if the scores are identical the consumer must be indifferent between the two bundles.

Utility Functions: Ordinal not Cardinal

- In order to give this score meaning, we invented a unit of measure called a “util”.
- This is the number of units of satisfaction a bundle gives a consumer, but in practice it only tells you about which bundle is preferred, not by how much a bundle is preferred.
  - I.e.: Suppose we have bundle X and Y.
  - Suppose U(X)=10 and U(Y)=5.
  - X is preferred to Y, but X is NOT 2x as preferred as Y.
- We call this an ordinal, as opposed to cardinal, ordering.
- A utility function is ordinal.

Utility Functions $\iff$ Rational Preference Relations

- $U(X) \geq U(Y) \iff X \succsim Y$
- $U(X) = U(Y) \iff X \sim Y$
- These utility functions are **continuous** which means that small changes in a consumption bundle result in small changes in measured utility.

1 Preference Relation, Many Utility Functions

- Suppose your preferences for the number of pictures of puppies are such that you always prefer more pictures of puppies to fewer pictures of puppies and you have rational preferences over pictures of puppies.
- One valid utility function to represent your preference relation is simply $U(p) = p$.
- $U(p) = p + 42$ is another valid utility function because it will rank bundles with more pictures higher than bundles with fewer pictures.
- $U(p) = 2 \cdot p + 39$ is another valid utility function.
- **All** these utility functions represent the same preference relations.

Utility Functions & Rationality

- Let's review the 3 axioms of rational preferences and how they apply to utility functions.
  1. **Completeness**: We can always rank two bundles X and Y by comparing U(X) to U(Y).
  2. **Reflexive**: $U(X) = U(X)$.
  3. **Transitive**: If $U(X) > U(Y)$ and $U(Y) > U(Z)$, then $U(X) > U(Z)$.

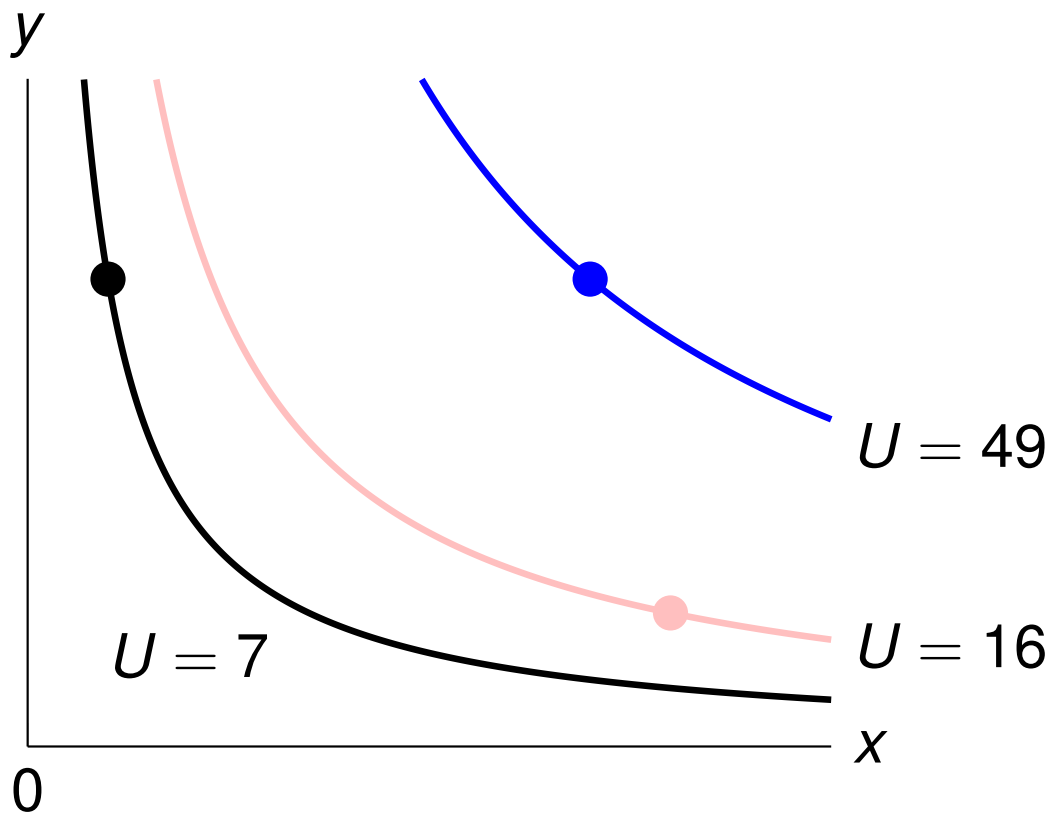
Indifference Curves Revisited

- Remember from 03 - Indifference Curves that all bundles on the same indifference curve are equally preferred.
  - The consumer is “indifferent” between any two bundles on the same indifference curve.
- If we are using an utility function to score bundles, then any two bundles on the same indifference curve must have the same utility value.
- So we can draw an indifference curve for a given utility function by plotting all bundles that have the same level of utility.
- All such indifference curves would fully represent a consumer’s preferences.

Indifference Curves Revisited

- Suppose $U(x, y) = xy$.
- Let's draw three indifference curves at U=7,16,49.
  - $1 = 2xy \implies y = \frac{7}{x}$.
  - $5 = 2xy \implies y = \frac{16}{x}$.
  - $10 = 2xy \implies y = \frac{49}{x}$.
- These are the same indifference curves as before in slides 03-Indifference Curves!

Indifference Curves Revisited



From Slides 3: The Slope of the Indifference Curve

- Look at any of the 3 indifference curves on the previous slide.
  - If we started at point x, and I took away 1 $x2$, then you would be less well-off since you prefer $x2$ and I took 1 away from you.
  - In order to keep you just as well-off, I could give you some $x1$ to compensate you.
  - Similarly, if I took away 1 $x1$, I could give you some $x2$ to compensate you.
  - This results in a negative slope of the indifference curve!

The MRS=Slope of Indifference Curves

- In the last set of slides we stopped at the intuition of the MRS, which captures the tradeoff a consumer **wants** to make between two goods.
- Now we want to calculate the Marginal Ratio of Substitution (MRS) mathematically.
- To do so, we will introduce/revisit **Marginal Utility**.
- Then, we will show that the ratio of the marginal utilities give us the MRS, and therefore the slope of the indifference curve.

Marginal Utilities

- If you get a 90 on the exam, you probably feel happy.
- If you got an extra point, a 91, you would feel a little bit happier.
- That extra happiness is what we call **Marginal Utility**.
- It is the change in your happiness/utility when you get one additional unit.
- That makes it sound like a calculus thing, and it is!

$$MU = \frac{\partial U(\cdot)}{\partial X}$$

Marginal Utilities

- Let's take our example of $U = xy$.
- $MU_x = \frac{\partial U}{\partial x} = y$.
- $MU_y = \frac{\partial U}{\partial y} = x$

Calculating MRS with Marginal Utilities

- Suppose we're thinking about Bill purchasing two goods x and y.
- Let $MU_x = 2$ and $MU_y = 4$.
- Suppose Bill decides to give up 1 unit of y. How many x's does he need to stay on the same indifference curve?
- **Answer**: He needs 2 of good x.
  - When Bill gave up that unit of y, he lost his $MU_y = 4$ utiles.
  - In order to get those 4 utiles back, he needs 2 of good x since each give Bill $MU_x = 2$ utiles.
- Or, his tradeoff, Bill's **MRS** is $\frac{-MU_y}{MU_x}$.
  - The MRS is negative because we are giving up (losing) one good in favor of (gaining) a number of the other good.

Questions for Class

1. What would a utility function look like for perfect substitutes preferences?

2. What would a utility function look like for perfect complements preferences?

3. What would the marginal utilities be for perfect substitutes and perfect complements?